

Exercise 58

For the following exercises, evaluate the expressions, writing the result as a simplified complex number.

$$\frac{3 + 2i}{1 + 2i} - \frac{2 - 3i}{3 + i}$$

Solution

Simplify the given expression.

$$\begin{aligned}\frac{3 + 2i}{1 + 2i} - \frac{2 - 3i}{3 + i} &= \frac{1 - 2i}{1 - 2i} \times \frac{3 + 2i}{1 + 2i} - \frac{2 - 3i}{3 + i} \times \frac{3 - i}{3 - i} \\ &= \frac{(1 - 2i)(3 + 2i)}{(1 - 2i)(1 + 2i)} - \frac{(2 - 3i)(3 - i)}{(3 + i)(3 - i)} \\ &= \frac{3 + 2i - 6i - 4i^2}{1 + 2i - 2i - 4i^2} - \frac{6 - 2i - 9i + 3i^2}{9 - 3i + 3i - i^2} \\ &= \frac{3 - 4i + 4}{1 + 4} - \frac{6 - 11i - 3}{9 + 1} \\ &= \frac{7 - 4i}{5} - \frac{3 - 11i}{10} \\ &= \frac{2(7 - 4i)}{10} - \frac{3 - 11i}{10} \\ &= \frac{2(7 - 4i) - 3 + 11i}{10} \\ &= \frac{14 - 8i - 3 + 11i}{10} \\ &= \frac{11 + 3i}{10} \\ &= \frac{11}{10} + \frac{3}{10}i\end{aligned}$$